SELF-SIMILAR SOLUTION TO THE PROBLEM OF THE EXPANSION OF A CYLINDRICAL COLUMN OF CONDUCTING GAS IN A LONGITUDINAL MAGNETIC FIELD

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The nonstationary radial motion of a long cylindrical column of conducting gas in a time-varying longitudinal magnetic field is considered. Exact solutions are found by the method of separating the variables for the system of equations of magnetohydrodynamics on the assumption that the statistical pressure of the plasma at the boundary of the column is proportional to the external magnetic pressure. Some numerical computations are performed and the energetic characteristics of the interaction process are calculated. The ratio of the useful work done by the gas over an infinite time interval to the initial energy of the column is given as a function of the magnetic Reynolds number. We note that a similar method was applied in [1], where not only was the average temperature taken over the cross section, but the inertia of the medium was also neglected. When the inertia is taken into account, we have the additional requirement that the statistical pressure be proportional to the magnetic pressure at the boundary of the column.

A physically similar model may be interpreted, for example, as the expansion of a compressible conducting gas column in a nonconducting incompressible fluid situated in a permeable cylinder of some radius R infinite along the axis of symmetry. The requirement that the statistical pressure be proportional to the magnetic pressure reduces to the condition that the external pressure on the boundary of the permeable cylinder of radius R should vary according to a specific law, which may easily be determined.

We shall make the following assumptions.

(1) The conductivity of the gas is finite and is determined by the temperature

$$\frac{\sigma}{\sigma_0} = \left(\frac{T}{T_0}\right)^n \qquad (n \geqslant 0). \tag{0.1}$$

(2) The gas is ideal; viscosity and thermal conductivity are not allowed for.

(3) Displacement currents are everywhere neglected. In particular, it is assumed that the variation of magnetic field strength on the external boundary of the expanding cylindrical column may be described by an arbitrary law, without considering the electromagnetic waves in the external nonconducting space. The latter assumption is correct if the velocity of expansion is much less than the velocity of light.

(4) A statistical pressure proportional to the external magnetic pressure is maintained on the external boundary of the column.

This requirement is connected with the condition that the problem be self-similar in the sense that the variables be separable.

1. Basic equations. In view of assumptions (1)–(3) the system of equations of magnetohydrodynamics in cylindrical coordinates has the form

$$\frac{\partial H}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(rvH) = \frac{c^2}{4\pi}\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{\sigma}\frac{\partial H}{\partial r}\right), \tag{1.1}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} = -\frac{1}{\rho}\frac{\partial}{\partial r}\left(p + \frac{H^2}{8\pi}\right), \qquad \frac{\partial \rho}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(rv\rho),$$

$$\rho c_v\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial r}\right) = -\rho p\frac{d}{dt}\left(\frac{1}{\rho}\right) + \frac{c^2}{16\pi^2\sigma}\left(\frac{\partial H}{\partial r}\right)^2 \qquad (p = R\rho T).$$

Here H (r, t) and v (r, t) are the longitudinal and radial components of the vectors **H** and **v**, respectively. The vectors **H** and **v** do not have any other components ($d/dz \equiv 0$, $d/d\varphi \equiv 0$). We will seek a solution satisfying the condition of proportional expansion, i. e.,

$$v(r, t) = \frac{r}{a(t)}\frac{da}{dt}, \tag{1.2}$$

where $a(t)$ is the unknown law of motion of the cylindrical column boundary.

We introduce the notation

$$h_1 = \frac{H}{H_0}, \quad p_1 = \frac{p}{H_0^2/8\pi},$$

$$\theta_1 = \frac{T}{T_0}, \quad \rho_1 = \frac{\rho}{H_0^2/8\pi RT_0}, \quad \lambda = \frac{a}{a_0},$$

$$\xi = \frac{r}{a}, \quad \tau = \frac{v_0 t}{a_0}, \quad t_0 = \frac{a_0}{v_0} = \frac{a_0}{\sqrt{RT_0}},$$

$$\nu = \frac{c^2 t_0}{4\pi\sigma_0 a_0^2}, \quad v_0 = \sqrt{RT_0} = \frac{\sqrt{\gamma RT_0}}{\sqrt{\gamma}}. \tag{1.3}$$

Here, in order to construct dimensionless quantities, we choose the following scales: H_0 is the magnetic field strength at the boundary of the column at the initial moment of time, a_0 is the initial radius of the column, T_0 is the temperature at the boundary of the column at the initial moment, v_0 is a characteristic velocity, σ_0 is the conductivity at a temperature T_0. We represent equations (1.1), (1.2), and (1.3) in the form

$$\frac{\partial}{\partial \tau}(\lambda^2 h_1) = \frac{\nu}{\lambda^2}\frac{1}{\xi^2}\frac{\partial}{\partial \xi}\left[\frac{\xi}{\theta_1^n}\frac{\partial}{\partial \xi}(\lambda^2 h_1)\right],$$

$$\rho_1\lambda''\lambda = -\frac{1}{\xi}\frac{\partial}{\partial \xi}(p_1 + h_1^2), \qquad \frac{\partial \rho_1}{\partial \tau} = -\frac{2\rho_1\lambda'(\tau)}{\lambda(\tau)},$$

$$\frac{\partial \theta_1}{\partial \tau} = -\varkappa p_1\frac{\partial}{\partial \tau}\frac{1}{\rho_1} + 2\nu\varkappa\frac{1}{\rho_1\theta_1^n}\frac{1}{\lambda^2}\left(\frac{\partial h_1}{\partial \xi}\right)^2, \qquad p_1 = \rho_1\theta_1,$$

$$\left(\varkappa = \frac{R}{c_v} = \frac{c_p}{c_v} - 1\right). \tag{1.4}$$

The third equation of system (1.4) may be integrated. We obtain

$$\rho_1 = \Phi(\xi) / \lambda^2(\tau). \tag{1.5}$$

Here $\Phi(\xi)$ is some function of ξ. Employing (1.5) and introducing the new unknown functions

$$h(\xi, \tau) = \lambda^2(\tau) h_1(\xi, \tau),$$

$$\theta(\xi, \tau) = \lambda^{2\varkappa}\theta_1(\xi, \tau), \tag{1.6}$$

the remaining equations may be written in the form

$$\frac{\lambda''}{\lambda(\tau)} = -\frac{1}{\xi\Phi(\xi)}\frac{\partial}{\partial\xi}\left(p_1 + \frac{h^2}{\lambda^4}\right), \qquad \frac{\partial\theta^n}{\partial\tau} = \frac{2n\nu\varkappa}{\lambda^{6-2\varkappa n}}\frac{1}{p_1}\left(\frac{\partial h}{\partial\xi}\right)^2,$$
$$\frac{\partial h}{\partial\tau} = \frac{\nu}{\lambda^{2-2\varkappa n}}\frac{1}{\xi}\frac{\partial}{\partial\xi}\left(\frac{\xi}{\theta^n}\frac{\partial h}{\partial\xi}\right), \qquad p_1 = \frac{\Phi(\xi)\,\theta}{\lambda^{2+2\varkappa}}. \qquad (1.7)$$

We are looking for the particular solution of system (1.7) in the form

$$h(\xi,\tau) = T(\tau)Z(\xi), \quad \theta^n(\xi,\tau) = V(\tau)X(\xi), \quad p_1(\xi,\tau) = G(\tau)Y(\xi) \qquad (1.8)$$

(i.e., the self similar solution [1]); here we shall assume that

$$T(0) = 1, \quad V(0) = 1, \quad G(0) = 1. \qquad (1.9)$$

It is easy to see that the variables in equations (1.7) may be separated on condition that

$$\frac{T^2(\tau)}{\lambda^4(\tau)\,G(\tau)} = \text{const} = 1. \qquad (1.10)$$

Here the constant is equal to 1 in view of (1.9) and the initial condition

$$\lambda(0) = 1. \qquad (1.11)$$

It follows from (1.10) that in this case the ratio of the statistical pressure to the magnetic pressure for every given particle is a constant quantity independent of time. This condition is fulfilled if a pressure is maintained on the external boundary of the cylindrical column which is proportional to the magnetic pressure (assumption (4)).

After setting (1.8) in system (1.7), employing condition (1.10) and separating the variables, the following two systems of equations are obtained:

for the function $T(\tau)$, $V(\tau)$, $G(\tau)$, $\lambda(\tau)$

$$\frac{\lambda''(\tau)}{\lambda(\tau)\,G(\tau)} = \alpha, \qquad \frac{V(\tau)\,\lambda^{2-2\varkappa n}}{T(\tau)}\frac{dT}{d\tau} = \beta,$$
$$\frac{\lambda^{6+2\varkappa n}\,G(\tau)}{T^2(\tau)}\frac{dV}{d\tau} = \mu, \qquad \frac{\lambda^{2+2\varkappa n}\,G(\tau)}{V^{1/n}(\tau)} = \psi, \qquad (1.12)$$

for the functions $X(\xi)$, $Y(\xi)$, $Z(\xi)$, $\Phi(\xi)$

$$-\frac{1}{\xi\Phi(\xi)}\frac{d}{d\xi}\left[Y(\xi) + Z^2(\xi)\right] = \alpha,$$
$$\frac{\nu}{Z(\xi)}\frac{1}{\xi}\frac{d}{d\xi}\left[\frac{\xi}{X(\xi)}\frac{dZ}{d\xi}\right] = \beta,$$
$$\frac{2n\nu\varkappa}{Y(\xi)\,X(\xi)}\left(\frac{dZ}{d\xi}\right)^2 = \mu, \qquad \frac{\Phi(\xi)}{Y(\xi)}X^{1/n}(\xi) = \psi. \qquad (1.13)$$

Here α, β, μ and ψ are some constant quantities. In view of the scaling quantities adopted and the normalizing conditions (1.9) for functions of τ the boundary conditions for the spatial functions will be as follows:

$$Z(\xi)|_{\xi=1} = 1, \quad X(\xi)|_{\xi=1} = 1, \quad Y(\xi)|_{\xi=1} = q, \qquad (1.13)$$

where q denotes the ratio of the statistical pressure to the magnetic pressure at the boundary of the column under consideration. On the basis of (1.19) the constant ψ from the last equations of (1.12) and (1.13) must be set equal to unity.

2. Integration of the resulting systems of equations. The unknown functions must satisfy not only the systems of equations (1.12), but also the additional condition (1.10), necessary for obtaining the particular solution under consideration, and so the constants α, β, and μ may not be arbitrary. In fact, we shall first of all consider the system of equations for functions of the variable τ. We will replace the function $G(\tau)$ in all the equations of (1.12), expressing it in terms of $\lambda(\tau)$ and $T(\tau)$ from (1.10).

We shall then have the system of four equations

$$\lambda''(\tau)\,\lambda^3(\tau) = \alpha T^2(\tau), \qquad \frac{d\ln T}{d\tau} = \frac{\beta}{\lambda^{2-2\varkappa n}}\frac{1}{V(\tau)}$$
$$\frac{dV}{d\tau} = \frac{\mu}{\lambda^{2-2\varkappa n}}, \qquad T(\tau) = V^{1/2n}(\tau)\,\lambda^{1-\varkappa}(\tau) \qquad (2.1)$$

for the unknowns $T(\tau)$, $V(\tau)$, and $\lambda(\tau)$.

The number of equations is larger than the number of unknowns and so the system can be consistent only if we have functions determining the relations between the constants α, β, and μ. In order to obtain these relations we shall consider the last three equations of the system (2.1); we have

$$T(\tau) = [\lambda(\tau)]^{\frac{(1-\varkappa)2n\beta}{2n\beta-\mu}}, \qquad V(\tau) = [\lambda(\tau)]^{\frac{(1-\varkappa)2n\mu}{2n\beta-\mu}}, \qquad (2.2)$$

$$\frac{(1-\varkappa)\,2n}{2n\beta-\mu}\lambda'(\tau) = [\lambda(\tau)]^{\frac{2n\beta(2\varkappa n-1)+\mu(1-2n)}{2n\beta-\mu}}. \qquad (2.3)$$

We differentiate the last equation and write it in the form

$$\lambda''(\tau) = \frac{2n\beta-\mu}{[(1-\varkappa)\,2n]^2}\left[2n\beta(2\varkappa n-1) + \mu(1-2n)\right][\lambda(\tau)]^{\frac{2n\beta(4\varkappa n-3)+\mu(3-4n)}{2n\beta-\mu}}. \qquad (2.4)$$

Setting $T(\tau)$ from (2.2) in the first equation of system (2.1), we obtain another equation for $\lambda(\tau)$:

$$\lambda''(\tau) = \alpha\lambda^{\frac{4n\beta(1-\varkappa)}{2n\beta-\mu}-3}. \qquad (2.5)$$

For system (2.1) to be self-consistent equations (2.4) and (2.5) must be identical. Two cases must be considered separately.

In the first case $\alpha \neq 0$. We equate the indices of powers of λ, and also the constant multipliers on the right sides of equations (2.4) and (2.5).

We obtain as a result

$$\beta = \frac{\mu}{\varkappa + 2\varkappa n - 1}, \qquad \alpha = -\mu^2\frac{2n+1}{4n^2(\varkappa + 2\varkappa n - 1)^2}. \qquad (2.6)$$

Here in deriving the expression for α use was made of the expression for β.

Thus, of the three constants α, β, and μ only one is independent. From equation (2.3) and conditions (1.11) and (2.6) we have

$$\lambda(\tau) = \left[1 + \frac{(n+1)\mu}{n(\varkappa + 2\varkappa n - 1)}\tau\right]^{\frac{2n+1}{2(n+1)}} \times \left(\mu = k\,\frac{2n(\varkappa + 2\varkappa n - 1)}{2n+1}\right). \qquad (2.7)$$

It follows from relations (2.2) that

$$T(\tau) = \lambda^{\frac{2n}{2n+1}}, \qquad V(\tau) = \lambda^{\frac{2n(\varkappa+2\varkappa n-1)}{2n+1}}. \qquad (2.8)$$

The independent constant μ is expressed in terms of the dimensionless initial velocity $k = \lambda'(0)$.

The second case is $\alpha = 0$. For the equations (2.4) and (2.5) to be identical it is necessary that the constant multiplier on the right side of equation (2.4) be equal to zero. There are two possibilities.

The first, $2n\beta - \mu = 0$, leads to the trivial solution $\lambda'(\tau) \equiv 0$. The second, $2n\beta(2\varkappa n - 1) + \mu(1 - 2n) = 0$, gives

$$\beta = \frac{(2n-1)\mu}{2n(2\varkappa n - 1)}. \qquad (2.9)$$

Here

$$\lambda(\tau) = 1 + k\tau, \qquad k = \frac{\mu}{2\varkappa n - 1}, \qquad (2.10)$$

$$T(\tau) = \lambda^{\frac{2n-1}{2n}}, \qquad V(\tau) = \lambda^{2\varkappa n - 1}. \qquad (2.11)$$

Thus the system of equations for functions of τ is fully solved.

It follows from (2.8) and (2.11) that the solution which has been obtained is characterized by the fact that the external magnetic field (i.e., the magnetic field on the external surface of the column) does not remain constant but decreases as the radius of the column increases. In fact,

$$h_1(1, \tau) = \frac{T(\tau)}{\lambda^2(\tau)} = \begin{cases} \lambda^{\vartheta} \text{ where } \alpha = 0 \ (\vartheta = -(2n+1)/2n), \\ \lambda^{\theta} \text{ where } \alpha \neq 0 \ (\theta = -(2n+2)/(2n+1)). \end{cases}$$

3. Solution of the system of equations (1.13) for the spatial functions. The first case is $\alpha = 0$. From the first equation of system (1.13) with $\alpha = 0$ and boundary conditions (1.14) we have

$$Y(\xi) = q_1 - Z^2(\xi), \qquad q_1 = 1 + q. \qquad (3.1)$$

From the third equation of system (1.13), employing (3.1), we obtain

$$X(\xi) = \frac{2n\nu\varkappa}{\mu}\frac{1}{Y(\xi)}\left(\frac{dZ}{d\xi}\right)^2 = \frac{2n\nu\varkappa}{\mu}\frac{1}{q_1 - Z^2}\left(\frac{dZ}{d\xi}\right)^2. \qquad (3.2)$$

We set this expression in the second equation of system (1.13); using (2.9) we obtain

$$Z_{\xi\xi}'' + B\frac{Z}{q_1 - Z^2}(Z_\xi')^2 - \frac{1}{\xi}Z_\xi' = 0, \qquad \left(B = 2 + \frac{(2n-1)\varkappa}{2\varkappa n - 1}\right). \qquad (3.3)$$

We have boundary conditions for this equation in the form

$$Z(1) = 1, \qquad \left.\frac{dZ}{d\xi}\right|_{\xi=1} = \left(\frac{\mu}{2n\nu\varkappa}q\right)^{1/2} = \left(\frac{k(2\varkappa n - 1)}{2n\nu\varkappa}q\right)^{1/2}. \qquad (3.4)$$

The second condition is obtained from the third equation of system (1.13) and the boundary conditions (1.14) for the functions $X(\xi)$, $Y(\xi)$.

To solve the equation (3.3) we introduce the new independent variable $x = \ln \xi$; we obtain for the function $u(x) = Z(\xi)$

$$u'' - 2u' + B\frac{u}{q_1 - u^2}(u')^2 = 0.$$

This equation does not contain the independent variable x explicitly. We thus introduce the new function $\varphi(u) = u'$; we obtain then for the unknown φ

$$\varphi' + B\frac{u}{q_1 - u^2}\varphi = 2, \qquad \varphi(1) = \left(\frac{k}{\nu}\frac{2\varkappa n - 1}{2\varkappa n}q\right)^{1/2}. \qquad (3.5)$$

Since $x = 0$ for $\xi = 1$, the boundary condition is obtained from (3.4):

$$u|_{x=0} = Z(\xi)|_{\xi=1} = 1, \quad u_x'|_{x=0} = \xi Z_\xi'|_{\xi=1} = \left(\frac{k}{\nu}\frac{2\varkappa n - 1}{2\varkappa n}q\right)^{1/2}.$$

The solution of equation (3.5) has the form

$$\varphi = (q_1 - u^2)^{\frac{B}{2}}\left[2\int_1^u \frac{du}{(q_1 - u^2)^{B/2}} + q^{-\frac{B}{2}}\left(\frac{k}{\nu}\frac{2\varkappa n - 1}{2\varkappa n}q\right)^{1/2}\right]. \qquad (3.6)$$

Since $\varphi = du/dx$, from (3.6) and the condition $u(0) = 1$ we find that

$$x = \frac{1}{2}\ln\left\{\frac{2q^{B/2}}{K}\int_1^u \frac{du}{(q_1 - u^2)^{B/2}} + 1\right\} \quad \left(K = \left(\frac{k}{\nu}\frac{2\varkappa n - 1}{2\varkappa n}q\right)^{1/2}\right). \qquad (3.7)$$

Hence we obtain the required solution of equation (3.3) satisfying conditions (3.4):

$$\xi = \left\{\frac{2q^{B/2}}{K}\int_1^Z \frac{d\omega}{[q_1 - \omega^2]^{B/2}} + 1\right\}^{1/2}. \qquad (3.8)$$

The ratio k/ν which enters in the expression (3.7) for K in fact represents the magnetic Reynolds number, since

$$\frac{k}{\nu} = \frac{4\pi\sigma_0 a_0 a'(t)|_{t=0}}{c^2} = R_m. \qquad (3.9)$$

Letting Z_0 represent the value of Z for $\xi = 0$, we have from (3.8), taking (3.7) into account,

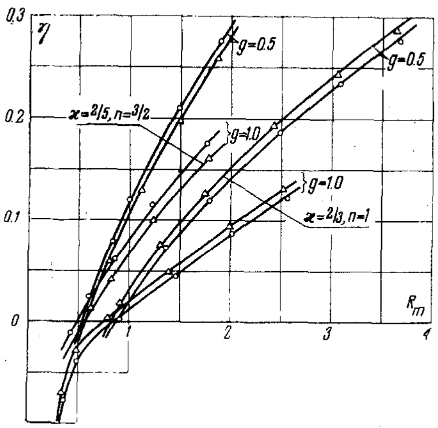
$$\int_{Z_0}^1 \frac{d\omega}{(q_1 - \omega^2)^{B/2}} = \frac{1}{2}\left(\frac{k}{\nu}\frac{2\varkappa n - 1}{2\varkappa n}\right)^{1/2} q^{\frac{1-B}{2}}, \qquad (3.10)$$

which gives the quantity Z_0 as a function of the magnetic Reynolds number R_m. Employing this relation, we may write the expression for Z (ξ) in the form

$$\xi = \left(1 - \frac{\psi(Z)}{\psi(Z_0)}\right)^{1/2} \left(\psi(Z) = \int_Z^1 f(\omega)\, d\omega,\ f(\omega) = \frac{1}{(q_1 - \omega^2)^{B/2}}\right). \quad (3.11)$$

Thus in the case when $\alpha = 0$ the system of equations for the spatial function may also be integrated completely [the solution for Y(ξ) is given by formula (3.1), for X(ξ) by formula (3.2) and the solution for Φ (ξ) is obtained from the fourth equation of system (1.13)].

The second case is $\alpha \neq 0$. In the system of equations (1.13) the constants α, β and μ, in accordance with (2.6) and (2.7), are unambiguously determined by specifying the dimensionless initial velocity $k = \lambda'(0)$. The system (1.13) cannot be integrated analytically and so we reduce it to a form suitable for numerical solution on an electronic computer.



To do this we express the functions Φ (ξ) and X(ξ) from the two last equations of system (1.13) in terms of Y (ξ) and Z (ξ); we have

$$X(\xi) = \frac{2n\nu\varkappa}{\mu} \frac{1}{Y(\xi)} \left(\frac{dZ}{d\xi}\right)^2,$$

$$\Phi(\xi) = \left[\frac{2n\nu\varkappa}{\mu}\right]^{-1/n} Y^{1+1/n}(\xi) \left(\frac{dZ}{d\xi}\right)^{-2/n}. \quad (3.12)$$

Setting these expressions in the first two equations of system (1.13), we obtain a system of two equations for the functions Y(ξ) and Z(ξ):

$$\frac{d}{d\xi}\left[\xi \frac{Y(\xi)}{Z_\xi'}\right] = \frac{2n\varkappa}{\varkappa + 2\pi\varkappa - 1} \xi Z$$

$$\frac{d}{d\xi}[Y + Z^2] = \frac{(2n+1)(2n\nu\varkappa)^{-1/n}}{4n^2(\varkappa + 2\varkappa n - 1)^2} \mu^{2+1/n} \xi Y^{1+1/n} (Z_\xi')^{-2/n}. \quad (3.13)$$

We carry out the change of independent variable $x = \xi^2$ and reduce the equations (3.13) to the form

$$\frac{d}{dx} \frac{Y}{Z_x'} = AZ, \qquad \frac{d}{dx}[Y + Z^2] = D x^{-1/n} Y^{1+1/n} (Z_x')^{-2/n},$$

$$A = \frac{2n\varkappa}{\varkappa + 2n\varkappa - 1},$$

$$D = k^2 (2n+1)^{-(n+1)/n} \left(\frac{k}{\nu}\right)^{1/n} \left(\frac{\varkappa + 2\varkappa n - 1}{\varkappa}\right)^{1/n} 2^{-\frac{n+2}{n}}. \quad (3.14)$$

We introduce the new unknown functions

$$\psi(x) = \frac{Y(x)}{Z_x'}, \qquad \varphi(x) = Z_x'. \quad (3.15)$$

We obtain the system of equations

$$\frac{d\psi}{dx} = AZ, \qquad \frac{dZ}{dx} = \varphi,$$

$$\frac{d\varphi}{dx} = D\left(\frac{\psi}{x}\right)^{1/n} \varphi^{(n-1)/n} - Z\frac{\varphi}{\psi}(2 + A) \quad (3.16)$$

for the unknown functions ψ (x), Z (x), φ (x) from equations (3.14).

On the basis of (1.14) and (3.4) we have the following boundary conditions:

$$Z(x)|_{x=1} = 1, \quad \varphi(x)|_{x=1} = \frac{K}{2},$$

$$\psi(x)|_{x=1} = \frac{2q}{K} \quad \left(K = \left(\frac{k}{\nu} \frac{2\varkappa n - 1}{2\varkappa n} q\right)^{1/2}\right) \quad (3.17)$$

for the required functions.

Thus finding the spatial function Z (ξ), Y (ξ), Φ (ξ) and X (ξ) in the case when $\alpha \neq 0$ is reduced to integrating system (3.15) together with conditions (3.17).

Some numerical computations were performed and the energetic characteristics of the interaction process were calculated, i.e., the amount of work performed when the column expands against the electric body forces, Joule losses inside the conducting gas, variation of internal and kinetic energy. By way of example, the figure gives some values of the coefficient η as a function of the magnetic Reynolds number (circles correspond to the value $k = 1.0$, triangles to $k = 0.5$). The coefficient η is specified as the ratio of the useful work done in the time from $t = 0$ to $t = \infty$ to the initial energy of the column, i.e.,

$$\eta = \frac{A_\infty - Q}{W_0 + U_0}$$

where A_∞ and Q_∞ are the work done against the electric body forces, and the magnitude of the Joule losses over the specified time interval, while W_0, U_0 are respectively the kinetic and internal energies at the initial moment of time.

The expressions for the magnitudes of the energies A_∞, Q_∞, W and U are not given here, since they may easily be obtained from the meaning of these quantities.

The functions obtained for other values of the parameters ϰ, n, q, k in both cases ($\alpha = 0$, $\alpha \neq 0$) are similar to those given above, except that for small q the allowable interval of variation of R_m (for which $0 \leq Z_0 \leq 1$) lies in the region of large values, and so the values of η are positive everywhere in this interval.

It is clear from the curves given that for some values of R_m the difference $A_\infty - Q_\infty$ becomes negative, although the work A_∞ performed against the electric body forces is here positive. A similar phenomenon was obtained in [2] for the case when there is no magnetic field inside the column at the initial moment of time, and the magnetic field strength at the boundary of the column is not equal to zero.

The data given show that a similar phenomenon may also occur in the case where there is a continuous initial magnetic field distribution inside and on the boundary of the column (in this case the initial distribution of the magnetic field is determined by the function Z (ξ)).

It must be stressed that in this paper, just as in [2], all the energy quantities refer to a time interval beginning from some "initial" moment, and we do not consider the way that this initial state comes about nor its energy characteristics.

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